Proposal for Higher Order Mode (HOM) Studies in SNS SC Cavities
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I. Start with cavity design, for each beta, developed by optimizing everything but HOM properties.

II. Calculate HOM frequencies, R/Qs, and nominal field profiles.
  A. 0, 1, 2, and 3 θ modes[1]
  B. Use conical shorts[2] on beam pipes outside ends of beam tubes, length = 10*beam pipe radius
  C. Include all frequencies up to 4 times the fundamental frequency[3]

III. For the frequency spreads of the HOMs due to manufacturing tolerances, use $\sigma/(\Im f-f_o\Im) = 0.0011$[4], where σ is the standard deviation of frequency spreads of a particular HOM, f is the average frequency of that HOM, and f_o is the fundamental frequency.

IV. Determine rms displacements of the beam from the cavity axes as a function of position along the linac, based on assumed quadrupole alignment errors, steering errors, and cavity position displacement errors, without considering the effects of HOMs.

V. Determine backward-wave breakup thresholds.
  A. This type of instability is caused by an off-nominal bunch position at the downstream end of a cavity adding energy to a HOM in that cavity, and that energy causing enough kick of subsequent bunches at the upstream end of the cavity that their downstream displacements allow the energy in the mode to grow. The phenomenon can be either longitudinal or transverse.
  B. For each HOM, initially assume a Q value equal to the BCS Q_o value at 2.1K.
    1. The BCS Q_o value scales as f^{-2}
  C. Evaluate each cavity along the linac, one at a time
    1. Determine the transit time factor and specific shunt impedance[5] of the mode at that location
    2. Consider cases where the cavity HOM has initial excitation but the entering beam has no errors
    3. Consider both longitudinal and transverse effects.
    4. Consider all mode frequencies within $\pm$ 6 σ (random) and $\pm$ 50 MHz (systematic) of the nominal mode frequency.
    5. Determine whether or not there are any HOMs for which the mode energy grows, rather than damps, with time.

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[1] Higher multipoles are not important based on past experience because of their low electric field concentrations near the beamline.
[2] Used to provide a defined boundary condition without appreciable artificial shunt impedance. Mesh needs to be “elastic” to fit cone, or steps will introduce artificial impedance.
[3] R/Q values above this frequency fall off sharply because $\lambda/(2*\pi)$ is small compared to the iris radius.
[4] From a least squares fit to the results of Cornell SRF-830102, which reports results for four nominally identical copper cavities, tuned to have the same fundamental frequency. These cavities were made expressly for the purpose of determining the reproducibility of the manufacturing technique used for Nb cavities.
[5] 0 θ modes are measured in ohms per meter, and are independent of Q_o. 1 θ modes can be measured in ohms per cubic meter, i.e., the longitudinal impedance they exhibit is proportional to the square of the bunch distance from the axis. 2 θ and 3 θ modes are also measured in ohms per cubic meter, and are based on the 1 θ component at a probable beam centroid distance off axis.

6. If any cases are found where the mode frequency grows, find the highest Q_o value at which this is no longer the case.

VI. Determine cumulative breakup thresholds.

A. This type of beam breakup results from amplification of random variations in the longitudinal and transverse positions of successive beam bunch centroids entering the superconducting portion of the linac. The effect becomes worse with increasing distance downstream and with increasing time from the beginning of the beam pulse.

1. This was first observed at SLAC as a transverse phenomenon
2. For v < c, the phenomenon can presumably also occur in the longitudinal direction

B. For each HOM, initially assume a Q value equal to the BCS Q_o value at 2.1K

C. Perform 1000 Monte Carlo runs for each HOM

1. Assume a realistic transverse position and longitudinal position noise spectrum at the entrance to the SC part of the linac.
2. For each run, assume a set of HOM frequencies, using the manufacturing spread discussed above.
3. Track the beam behavior as a function of time and distance along the linac
   a. If the bunch length is of the same order of magnitude as the wavelength, divide the bunch into macroparticles
4. If the beam growth due to the mode is unacceptable in any of the 1000 runs, reduce the assumed Q until this just ceases to be the case.

VII. Establish Q limits for each HOM

A. For each HOM, select the Q limit which is the lower of the two found in V. and VI. above

B. Reduce this another factor of 5 as a safety margin[6].

C. If any of the required Qs are unrealistically low due to low fields in the end cells with the nominal cell shape, go directly to step X.

VIII. Search for trapped modes.

A. Assume that the actual surface location in a cavity will fluctuate about 0.5 mm from the design value, but that the cavity will retain its cylindrical symmetry[7].

B. Assume that the actual surface location deviation can be represented by a Fourier series, with distance measured along the surface from one end of the cavity to the other.

C. Perform 100 Monte Carlo runs for each HOM

1. Define 200 Fourier coefficients for sine waves, and 200 for cosine waves
2. For each run, select a coefficient randomly with $\sigma = 0.025$ mm
3. For each run, compute the cavity field profile
4. Compare this field profile with that for the nominal case

D. For 0 θ modes, multiply the Q found in VII.B. by the ratio of the lowest sum of both end cell energies from the Monte Carlo runs to the sum of both end cell energies in the nominal case.

E. For 1, 2, and 3 θ modes, multiply the Q found in 7B by the lower of the two ratios of the end cell energy from the Monte Carlo runs to the end cell energy for the corresponding cell in the nominal case.

IX. Develop damping probes which achieve the damping levels identified in VIII. D. and E. above

X. If IX cannot be achieved for one of the two beta values, modify the cell shape so that it becomes feasible.

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6 This safety factor allows for a factor of 2 uncertainty in the calculation compared to reality, for simultaneous action by multiple HOMs, and for higher shunt impedances in some partially trapped HOMs than in the nominal field profile of the same mode.

7 This assumption is necessary due to computational limitations.

XI. Determine the amount of HOM power to be handled by each HOM coupler.